Estimation of the Stage Efficiency of Simple, Agitated Vessels Used in Mixer-settler Extractors

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A method of calculation is presented by which estimates may be made of the stage efficiency of continuously operated, agitated, baffled vessels used in mixer-settler extractors. The calculations are limited to cases where the agitating impeller is a flat-blade turbine, and do not include estimates of the entrance and exit effects. The method has been tested with all the available experimental data, which include three different sizes of vessels, systems, and impeller sizes, and a variety of operating conditions including speeds of agitation, rates of flow, and ratios of contacted liquids. Because of limitations of the author's knowledge, the calculations are necessarily approximate, but they nevertheless correctly indicate the nature of the variations in stage efficiency to be expected with all of the design and operating variables for which tests could be applied.

It is the purpose of this paper to gather into one place the theoretical principles and available correlations which will permit the estimation of the stage efficiency of mixer-settler extractors, as far as this can be done with the presently available knowledge. It will become evident in the course of this development that there are severe limitations to our knowledge. The methods outlined here are therefore not expected to supplant experimental work. But they will serve to indicate the following:

(1) How rough estimates of stage efficiencies can be made in the absence of any experimental work,
(2) What will be the effect of changing any of the common operating variables such as agitator speed, impeller diameter, rate of flow, flow ratio, and the like, and
(3) Where additional fundamental work must still be done.

Mixer-settler extractors are single-stage devices in which the liquid to be extracted is intimately mixed with the extracting solvent for a period of time to allow a reasonable approach to equilibrium, whereupon the insoluble extract and raffinate solutions are permitted to settle from each other and are mechanically separated. In the most important cases operation is continuous, and the mixing and settling are done in separate vessels. The limitations of the author's knowledge, as developed below, make it necessary to confine this discussion to extraction in baffled, agitated vessels used as *mixers*, where the agitating impeller is a flat-blade turbine, centrally located.

NATURE OF DISPERSIONS

When two insoluble liquids are contacted in an agitated vessel, one becomes dispersed in the form of small droplets (the dispersed phase) which are suspended in a continuum of the other (the continuous phase). It has been observed that it is ordinarily difficult to obtain dispersions containing more than approximately 75% by volume of dispersed liquid, or less than 25% continuous phase. It is believed that this is true because the small droplets which form are substantially spherical, and because the void space in a closely packed bed of spheres is 26% by volume. Therefore in extractions involving ratios of liquids of 1:3 or less by volume, the liquid present in the minority will probably be the dispersed phase.

In the range of liquid ratios from 1:3 to 3:1, either liquid may be dispersed. Within this range, at least for batch operations, it has ordinarily been found that that liquid will be continuous in which the agitator impeller would be immersed if the agitator and the liquids were at rest. If the impeller happens to be located at the position of the interface between the liquids when at rest, there is no way to predict with certainty which liquid will be dispersed. In such cases it has sometimes been found that one liquid will be dispersed at low impeller speeds, the other at high (*15*). In batch experiments with 16 organic liquids contacted with equal volumes of water in a baffled vessel fitted with a centrally located, flat-blade turbine impeller, the water was in every case the continuous phase (*15*). With continuous flow, these generalizations need not necessarily apply and the method of introducing the liquids into the vessel may exert an additional influence.

Mass-transfer Rates

The liquid to be extracted is contacted with the extracting solvent in order to bring about a mass transfer of the dissolved solute in the former to the latter. If the usual two-film theory is accepted, the rate of approach to equilibrium distribution of the dissolved solute between the two liquids is controlled by the mass-transfer resistances residing within both the continuous and dispersed phases. The concentrations of the solute in the liquids at the interface between the liquids are assumed to be those corresponding to equilibrium.

The Continuous Phase

Continuous-Phase Mass-Transfer Coefficients. The mass-transfer resistance within the continuous phase is measured by a mass-transfer coefficient, k_C, which is in reality the reciprocal of the resistance.

Rushton (*16*) has suggested that for mass-transfer operations in agitated vessels, under conditions of fully developed turbulence and for batch operation;

$$N_{Sh} = K N_{Re}^{x} N_{Sc}^{s} \qquad (1)$$

A similar expression has also been successfully used in describing heat-transfer coefficients for various arrangements of heating surfaces in such vessels. The available data for establishing the constants of Equation (1) for present purposes are unfortunately meager.

Mack and Marriner (*10*) dissolved pellets of benzoic acid in aqueous caustic solutions, in baffled vessels under batch conditions. They showed that, although the rate of solution as measured in pounds per hour, for example, was influenced by the surface area and hence the particle diameter, the mass-transfer coefficient k_C was apparently independent of particle size. Their data can be put in the form of Equation (1), but for lack of information on the diffusivity of benzoic acid in the caustic solutions the constants cannot all be determined. It was noted that $x = 0.64$.

Oyama and Endoh (*14*) similarly dissolved sugar particles in a baffled vessel with a flat-blade turbine. Their data were correlated by plotting $N_{Sh}/N_{Sc}^{0.5}$ against the group $N_P^{1/3} N d^{5/3} d_p^{4/3} \rho / T\mu$ on logarithmic paper to give a reasonably good straight line, at least at high values of the latter group. Since, however, the value of N_P was not constant at the speeds of rotation used, the correlation cannot be put in the form of Equation (1).

Johnson and Huang (*5*) dissolved rings of soluble solid, which were cast into the bottom of the vessel, into liquids agitated in the vessel. In baffled vessels with a flat-blade turbine impeller, regardless of the position of the solid, a uniform mass-transfer coefficient was obtained, and this was correlated with the operating conditions by an expression in the form of Equation (1):

$$N_{Sh} = 0.0924 N_{Re}{}^{0.71} N_{Sc}{}^{0.5} \qquad (2)$$

Calderbank and Korchinski (*1*) measured the rate of heat transfer for liquid drops falling slowly through an unagitated continuous, insoluble liquid. They found that the continuous-phase heat-transfer coefficient for the liquid surrounding the liquid particles followed the data for solid spheres reasonably well up to droplet Reynolds numbers of 200. Although in the agitated vessel there appears to be no way of estimating the droplet Reynolds numbers, since the droplet size is small it is not unreasonable to expect that the continuous-phase mass-transfer coefficient k_C will be essentially the same for either a solid or a liquid dispersed phase. Theodore (*17*) contacted butyl alcohol saturated with water (dispersed phase) with water (continuous phase) in two sizes of baffled vessels, agitated with flat-blade turbines under continuous-flow conditions. After estimating the interfacial area by methods outlined below, it was found that Equation (2) described the values of k_C reasonably well.

There are several published works involving dissolution of solids in unbaffled vessels, but the circumstances described later limit considerations here to baffled vessels. Since there appear to be no other relevant researches, it is recommended that k_C be computed through Equation (2) until something more directly applicable is available.

Other Continuous-Phase Considerations. To compute the Schmidt number, a diffusivity is required. In the absence of experimental data, this may be estimated by the method of Wilke and Chang (*20*), which will also serve for dispersed-phase diffusivities.

For computing the impeller Reynolds number, the density and viscosity of two-phase liquid mixtures has been shown to be given by

$$\rho = \rho_D \phi_D + \rho_C \phi_C \qquad (3)$$

and

$$\mu = \frac{\mu_C}{\phi_C}\left[1 + \frac{1.5\phi_D \mu_D}{\mu_C + \mu_D}\right] \qquad (4)$$

for baffled vessels (*8, 19*).

For the subsequent treatment of the mass-transfer rates in the dispersed phase under continuous-flow conditions, it will be necessary to assume that the concentration of dissolved solute in the continuous phase is essentially uniform throughout the vessel at the effluent value. Karr and Scheibel (*6*) found this to be true in an unbaffled vessel. When the residence time for the continuous phase is reasonably large and especially when the vessel is baffled so that strong vertical currents are developed, this assumption does not seem unreasonable.

The Dispersed Phase

The droplets of dispersed phase are immersed in the continuous liquid under nonequilibrium conditions for the distributed solute, and they thus experience an unsteady-state diffusion of solute either inwardly or outwardly, depending upon whether the extract or the raffinate is dispersed. There have been no measurements of the rate of mass transfer under conditions even remotely approaching those of interest here. Consequently all of the reasoning must be by inference from related work, much of it having to do with heat transfer. The mass- and heat-transfer processes are, fortunately, analogous. But even the heat-transfer data were not taken under conditions resembling those in the agitated vessel.

For present purposes, holding time and drop diameter of the dispersed phase will be required.

Hold-up and Holding Time. The hold-up of the dispersed phase in the vessel is defined as the volume fraction ϕ_D of the total liquid in the vessel which is dispersed phase, under operating conditions. In the case of a batch operation, this will be the same as the volume fraction of the dispersed liquid in the batch.

In the case of a continuously operated vessel, ϕ_D is the same as the fraction of dispersed liquid in the feed mixture only under conditions of fairly vigorous agitation. At low power input to the agitator, ϕ_D will be less than the fraction of the dispersed liquid in the feed, at least in those cases where the less dense liquid is dispersed. No systematic study of this phenomenon has been made. The ratio of ϕ_D to volume fraction of dispersed liquid in the feed mixture is probably influenced by such physical properties of the liquids as densities, viscosities, and interfacial tension; rate of flow; shape and size of vessel; extent of baffling; type and position of impellers; and power developed by the impeller. The only available data are shown in Figure 1. Lack of knowledge of the nature of the influence of the many variables prevents better correlation.

Holding time, θ, is the average time of retention of the dispersed phase. In continuous operation,

$$\theta = \frac{V\phi_D}{v_D} \qquad (5)$$

Drop Diameter. The drop diameter d_p can be estimated from the dispersed-phase holdup and the interfacial area, a, assuming the particles to be spherical:

$$d_p = \frac{6\phi_D}{a} \qquad (6)$$

This is the diameter of a sphere whose volume/surface ratio is the same as

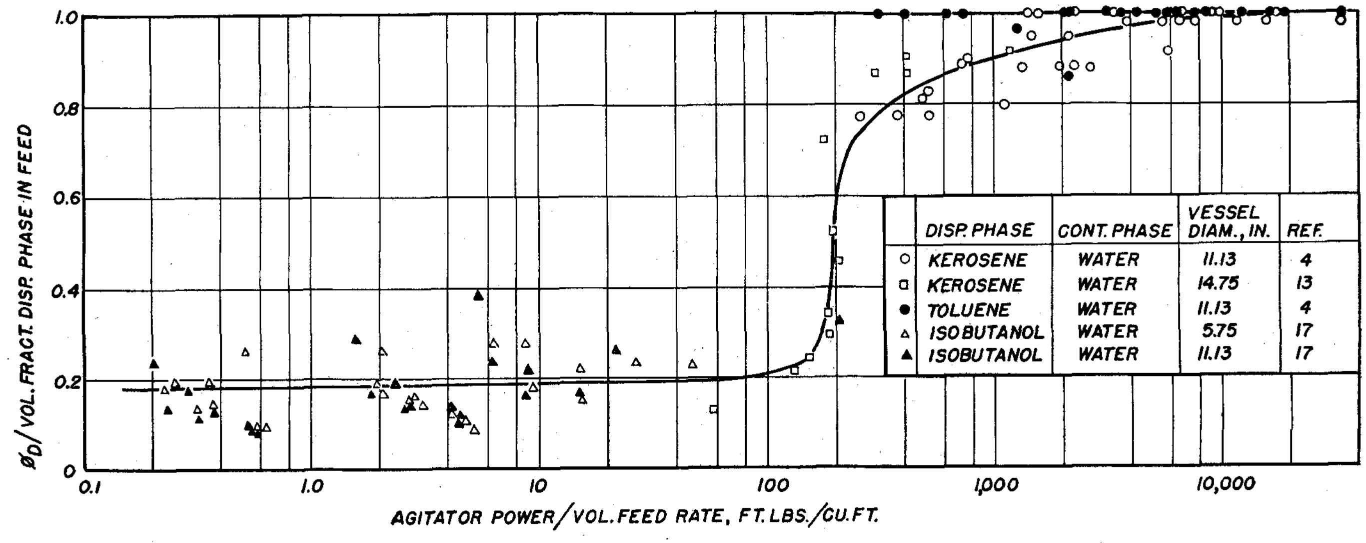

Fig. 1. Dispersion of organic liquids in water, baffled vessels agitated with flat-blade turbine impellers, with continuous, cocurrently upward flow of liquids.

that of all the dispersed phase in the vessel.

There have been two studies made of the interfacial surface of dispersed liquids in agitated vessels, using a light-scattering technique (*15*, *19*). The data of Rodger, *et al.* (*15*) appear to be the better correlated, although of a more limited nature. For dispersed-phase volume fraction $\phi_D = 0.50$, with a flat-blade turbine impeller centered in a baffled vessel operated batchwise, these authors offer two correlations. The first is the better, but it requires information on the break-time of the dispersion, and this will probably be unavailable under circumstances where calculations such as these will be made. The second, which is repeated here, is likely to be the more practical:

$$a_{50} = \frac{55}{T} (N_{We})^{0.25} \left(\frac{\nu_D}{\nu_C}\right)^{0.2} \cdot \exp\left(3.6 \frac{\Delta\rho}{\rho_C}\right) \exp\left(0.35 \frac{T}{T_0}\right) \quad (7)$$

In order to make the equation completely homogeneous dimensionally, a standard vessel diameter T_0 (0.5 ft. or equivalent in other length units) was included.

The correlation of Vermeulen, *et al.* (*19*) covers dispersed-phase volume fractions other than $\phi_D = 0.50$, although most of such data are for gas-liquid systems. It was shown that d_p is proportional to f, where f is a function of ϕ_D as given in Figure 2. Since $f = 2.0$ at $\phi_D = 0.50$ then

$$d_p = f\, d_{p50}/2.0 \quad (8)$$

In the absence of anything more immediately applicable, these batch measurements will be used for continuously operated vessels.

Unsteady-state Mass Transfer for Liquid Drops. The development here follows very closely one suggested by Calderbank and Korchinsky (*1*).

For rigid spheres, with initial uniform solute concentration c_D^0, and a constant solute concentration at the surface c_{Di} (which is tantamount to assuming no diffusional resistance in the continuous phase), of diameter d_p and time of exposure θ, the final average solute concentration c_D is given by (*12*)

$$\frac{c_D^0 - c_D}{c_D^0 - c_{Di}} = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \cdot \exp\{-4D_D\pi^2 n^2 \theta/d_p^2\} \quad (9)$$

Equation (9) has been shown to be empirically well represented by (*17*)

$$\frac{c_D^0 - c_D}{c_D^0 - c_{Di}} = (1 - \exp\{-4D_D\pi^2\theta/d_p^2\})^{0.5} \quad (10)$$

Kronig and Brink (*7*) have shown that for liquid spheres, with no continuous-phase mass-transfer resistance, internal circulation alters Equation (9) so that it becomes

$$\frac{c_D^0 - c_D}{c_D - c_{Di}} = 1 - \frac{3}{8} \sum_{n=1}^{\infty} A_{n^2} \cdot \exp\{-\psi_n(64)D_D\theta/d_p^2\} \quad (11)$$

Equation (11) theoretically is applicable only to very low sphere Reynolds numbers and zero interfacial tension, but Dankwerts (*2*) minimizes these limitations. Equation (*11*) has been shown to be very well represented empirically by (*1*)

$$\frac{c_D^0 - c_D}{c_D^0 - c_{Di}} = (1 - \exp\{-2.25(4)D_D\pi^2\theta/d_p^2\})^{0.5} \quad (12)$$

Comparison of Equations (10) and (12) indicates that the effective diffusivity for a liquid drop is $2.25D_D$. This has been confirmed in the case of heat transfer between liquid drops falling under the force of gravity through a surrounding liquid, where the average effective thermal diffusivity was 2.25 times the ordinary value (*1*). In a review of other data of this sort, Calderbank and Korchinski (*1*) show that this will be a satisfactory concept except where the liquid drops oscillate, which may increase the effective diffusivity (mass or thermal) by as much as 70-fold. But with the small droplets encountered in a mixing vessel, oscillation will probably not be encountered.

When there is a continuous-phase resistance to mass transfer, Equation (12) is unsatisfactory. In the case of heat transfer to rigid spheres, this problem has been solved, however, and presumably by replacing the heat-transfer terms by corresponding, analogous mass-transfer terms in the resulting relationships, and by using an effective diffusivity D_D' equal to 2.25 times the normal value, a reasonably satisfactory estimate can be made. The well known Gurney-Lurie charts are such heat-transfer solutions, and they appear in such standard texts as that of McAdams (*11*). These charts would permit computation of the solute concentration at time θ at any distance from the center of the drop, but since the final average concentration c_D is desired, a tedious integration would be required to complete the calculation.

Grober (*3*), however, has computed the heat-transfer case to give average final values directly. In terms of heat transfer, his work can be put in the form of a chart

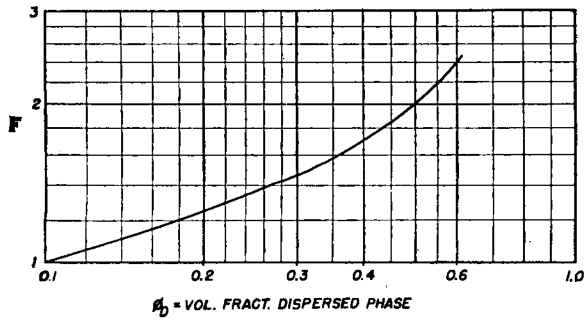

Fig. 2. Drop-size function (*19*).

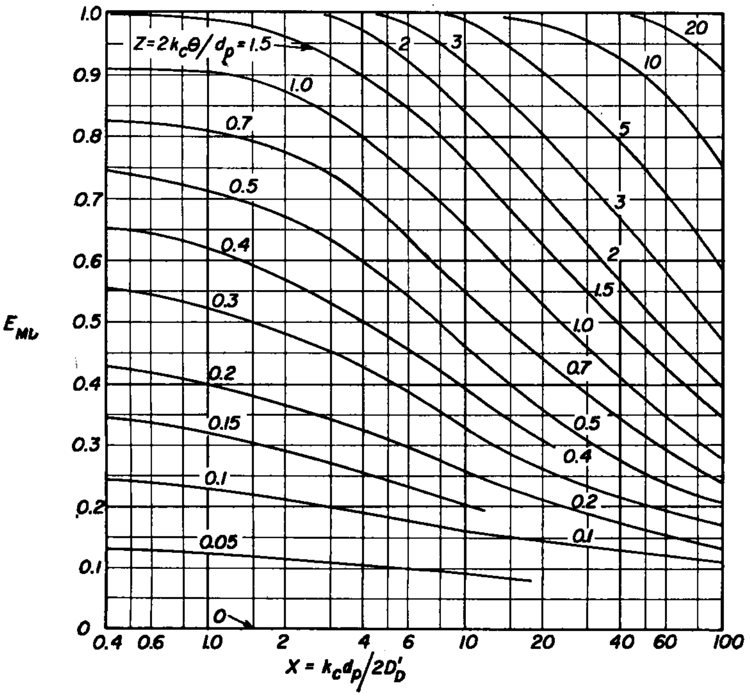

Fig. 3. Murphree dispersed-phase stage efficiency, adapted from Grobar (3).

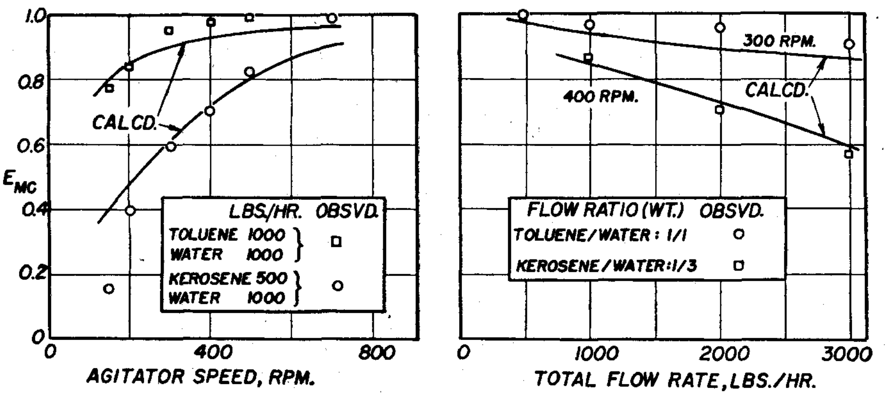

Fig. 4. Effect of agitator speed and total flow rate, extraction of benzoic acid from kerosene and toluene into water in continuously operated, baffled vessels (*4*). Vessel diameter = 5.75 in., impeller diameter = 2 in.

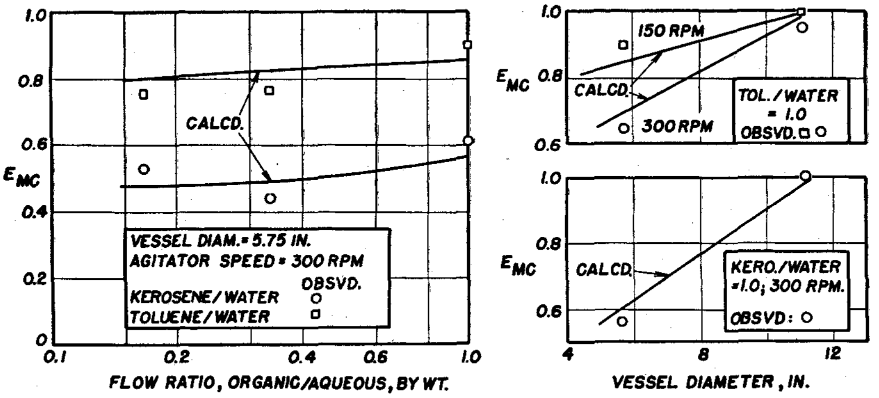

Fig. 5. Effect of phase ratio and vessel diameter, extraction of benzoic acid from kerosene and toluene into water in continuously operated, baffled vessels (*4*). Total liquid flow rate = 3000 lb./hr.

where the abscissa is $X = h_C r/k_H$ and the ordinate $(t_D^0 - t_D)/(t_D^0 - t_C)$, and a parameter is $Z = h_C\theta/C_P\rho_D r$. By noting that h_C can be replaced by k_C, t by c, and $k_H/C_P\rho_D$ by D_D', then

$$X = h_C r/k_H \approx k_C d_p/2D_D' \qquad (13)$$

$$\frac{t_D^0 - t_d}{t_D^0 - t_C} \approx \frac{c_D^0 - c_D}{c_D^0 - c_D^*} \qquad (14)$$

$$Z = \frac{h_C\theta}{C_P\rho_D r} = \left(\frac{k_H\theta}{\rho_D C_P}\right)\left(\frac{h_C}{k_H r}\right) \approx (D_D'\theta)\left(\frac{k_C}{D_D' r}\right) = \frac{2k_C\theta}{d_p} \qquad (15)$$

Further, it may be noted that, if c_D^* corresponds to equilibrium with the effluent continuous-phase concentration, then

$$\frac{c_D^0 - c_D}{c_D^0 - c_D^*} = E_{MD} \qquad (16)$$

where E_{MD} is the fractional Murphree dispersed-phase stage efficiency. Such a chart, with mass-transfer rather than heat-transfer notation, is shown in Figure 3. E_{MD} is the objective of the calculations herein discussed.

END EFFECTS

In addition to the extraction resulting from the operation of the agitator in the mixing vessel, there will be additional extraction occurring merely as a result of the continuous introduction of the two liquids into the vessel, which will depend upon the manner of introduction. Merely passing the liquids through the vessel with the agitator at rest will result in some extraction, although this will not necessarily give the same "entrance effect" as when the agitator is operating. There will be additional extraction during the disengaging of the liquids in the settler. Neither of these effects, together constituting the "end effects," can be estimated. An estimate of the stage efficiency as E_{MD} alone will therefore be conservative.

SAMPLE CALCULATION

Flynn and Treybal (*4*) present data on the extraction of benzoic acid from kerosene and from toluene into water in continuously operated, baffled vessels of two sizes, using flat-blade turbine impellers, centrally located. Their Run 30.6 will be used to illustrate the estimation of stage efficiency. The circumstances are:

Vessel diameter = T = 0.479 ft.
Vessel volume = V = 0.0865 cu. ft.
Impeller diameter = d = 0.1667 ft.
Agitator speed = N = 30,000 rev./hr. (500 rev./min.)
Rates of flow: kerosene, 500 lb./hr., or v_D = 9.78 cu. ft./hr.; water, 1500 lb./hr., or 24.1 cu. ft./hr.
Kerosene dispersed, water continuous
Densities: ρ_D = 51.1 lb./cu. ft., ρ_C = 62.2 lb./cu. ft.
Viscosities: μ_D = 6.1 lb./(ft.)(hr.)
μ_C = 2.30 lb./(ft.)(hr.)
Interfacial tension = 33 dynes/cm., σ = 950,000 lb./hr.^{2}

The calculations follow. D_C for benzoic acid-water = 0.0000380 sq. ft./hr., and N_{Sc} = 970 [Linton and Sherwood (*9*)]. D_D = 0.00001525 sq. ft./hr., estimated (*20*).

At the agitator speed used, ϕ_D will be the volume fraction kerosene in the feed mixture, or 9.78/(9.78 + 24.1) = 0.288. Therefore ϕ_C = 1 − 0.288 = 0.712. Equation (3): ρ = 59.1 lb./cu. ft.; Equation (4): μ = 4.25 lb./(ft.)(hr.) $N_{Re} = d^2N\rho/\mu$ = 11,600. Equation (2): $N_{Sh} = k_C T/D_C$ = 2230, whence k_C = 0.177 ft./hr.

$N_{We} = d^3N^2\rho_C/\sigma$ = 273; ν_D = 6.1/51.1 = 0.1193 sq. ft./hr., ν_C = 2.30/62.2 = 0.037 sq. ft./hr. T_0 = 0.5 ft., $\Delta\rho$ = 62.2 − 51.1 = 11.1 lb./cu. ft. Equation (7): a_{50} = 1640 sq. ft./cu. ft. At ϕ_D = 0.50, Equation (6): d_{p50} = 0.00183 ft. Figure 2: f = 1.5 at ϕ_D = 0.288. Equation (8): d_p = 0.00137 ft.

Equation (5): At ϕ_D = 0.288, θ = 0.00255 hr. D_D' = 2.25 (0.00001525) = 0.0000343 sq. ft./hr. Equation (13): $X = k_C d_p/2D_D'$ = 3.53. Equation (14): $Z = 2k_C\theta/d_p$ = 0.66. Figure 3: E_{MD} = 0.70, or a Murphree dispersed-phase stage efficiency of 70%. This is for the agitation effect only.

Flynn and Treybal (*4*) reported continuous-phase Murphree efficiencies, and to convert the above figure to this basis, the relation

$$E_{MC} = \frac{E_{MD}}{E_{MD} + \dfrac{mv_C}{v_D}(1 - E_{MD})} \qquad (16)$$

may be used. Here $m = dc_C/dc_D = dc_{water}/dc_{kerosene}$ = 0.2665 in weight/volume units for the reported equilibrium curve, and v_C/v_D = 24.1/9.78 = 2.47 cu. ft./cu. ft. Therefore, Equation (16): E_{MC} = 0.79 (calculated).

The reported E_{MC} is 0.914, but this includes the end effects. Flynn (*4*) indicated that at these rates of flow but for zero agitator speed, E_{MC} = 0.386. Even assuming that the end effects at 0 and at 500 rev./min. are the same, it was not at first clear how the end effects could be taken into account in order to provide a comparison between observed and calculated stage efficiency. It was finally decided that N_{toD} was most likely to be an additive quantity. For unitorm continuous-phase concentrations, the necessary relationships are

$$N_{toD} = -\ln(1 - E_{MD}) \qquad (17)$$

$$E_{MD} = 1 - 1/e^{N_{toD}} \qquad (18)$$

E_{MD} corresponding to E_{MC} = 0.386 is 0.30, by Equation (16), which in turn corresponds to an end effect N_{toD} (Equation (17)) = 0.36. Similarly the total N_{toD} corresponding to E_{MC} = 0.914 is 1.76, whence the N_{toD} due to the agitator is 1.76 − 0.36 = 1.40. The last corresponds [Equations (18) and (16)] to an agitator E_{MC} = 0.827 (observed). This is to be compared with the calculated value, 0.79.

OTHER COMPARISONS

From the total of 153 runs at other than zero agitator speed reported by Flynn (*4*), it is possible to pick those which illustrate the effect of particular variables, and the comparison of observed and calculated results are shown in Figures 4 and 5 on the same basis as the sample calculation. The runs chosen for these figures are those in which the experimental efficiencies are not all nearly 100%, and it is evident that the methods of computation are capable of following the observed trends very well. All the data for all the runs are included in the comparison of Figure 6, although since more than half of the observed measurements show efficiencies above 90%, most of the points are superimposed in the upper right-hand corner. The average error, $(\Delta E_{MC}/\text{observed } E_{MC})100$, for all the runs, is 21%. The low stage efficiencies, where the errors are largest, were observed at low agitator speeds corresponding to agitator Reynolds numbers of less than 5000. Turbulence is not fully developed at these speeds, and Equation (2) is very likely the principal source of calculation error here.

Overcashier, *et al.* (*13*) extracted butyl amine from kerosene with water in a 14.75-in. diam. baffled vessel, operated continuously, with flat-blade turbine impellers. The comparison of calculated and observed results are shown in Figure 7. These authors expressed their results in terms of the over-all countercurrent cascade efficiency. In order to preserve the same form for Figure 7 as was used by them to display their data, the calculated efficiencies were put in this form also, and the observed end effect was added to the calculated values. The scattering of points which appears in these graphs, at least for the calculated and probably for the observed data as well, is due to the rather erratic variations in dispersed-phase holdup reported. The average error between calculated and observed results, for all the runs, is 14.2%, calculated as before.

It is interesting to note that in the case of both Flynn's and Overcashier's data, when the volume of organic liquid exceeded 74%, it was necessary to assume the aqueous phase to be dispersed in order to get reasonable agreement between calculated and observed results, although neither set of authors reported this as an observation.

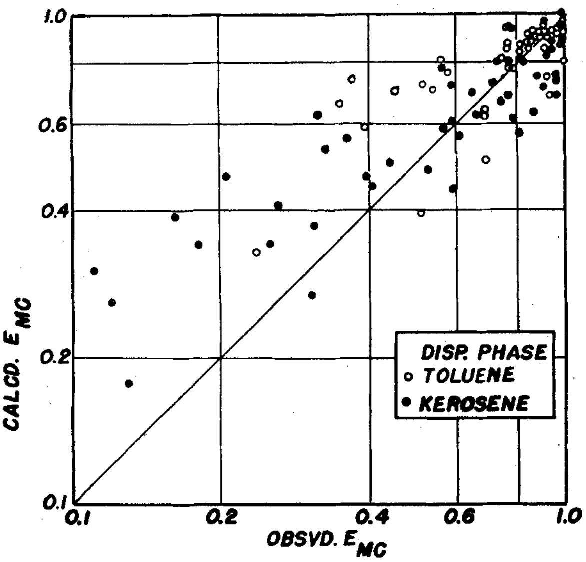

Fig. 6. Comparison of calculated and observed results for all runs, extraction of benzoic acid from kerosene and toluene into water in continuously operated, baffled vessels (*4*).

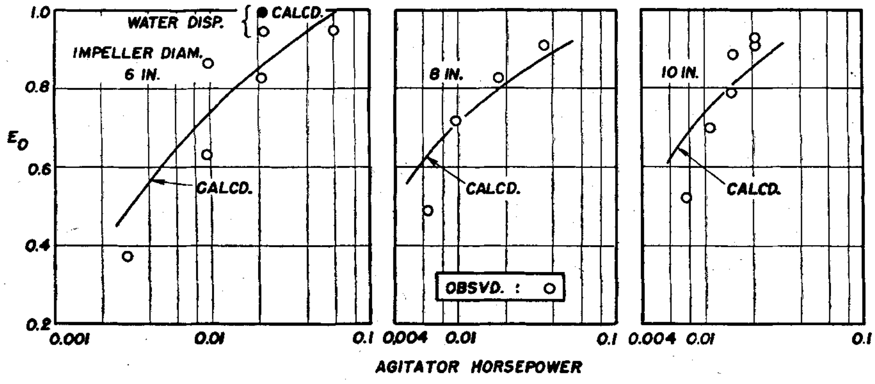

Fig. 7. Effect of agitator power and impeller diameter, extraction of butyl amine from kerosene into water in a continuously operated, baffled vessel (*13*). Vessel diameter = 14.75 in.; flow rates 7.5 gal./min. kerosene, 4.78 gal./min. water.

CONCLUSIONS

The methods of computation outlined herein permit rough estimates to be made of extraction stage efficiencies for baffled vessels agitated with flat-blade turbine impellers, and are capable of predicting the general nature of the influence of the principal variables involved in the design and operation of these devices. End effects cannot be estimated, and the total extraction to be expected will be somewhat greater than that calculated without allowance for these. Better agreement between calculated and observed results can be expected as more directly applicable experimental data accumulate.

NOTATION

Use any consistent set of units. English units are given as examples.

a = specific interfacial area, sq. ft./cu. ft.
a_{50} = specific interfacial area at $\phi_D = 0.50$, sq. ft./cu. ft.
A_n^2 = eigen value
c_D = average solute concentration in a drop, lb. moles/cu. ft.
c_D^0 = initial uniform solute concentration in a drop, lb. moles/ cu. ft.
c_{Di} = concentration of solute at the surface of a drop, lb. moles/cu. ft.
c_D^* = solute concentration in a drop which is in equilibrium with the solute concentration in the continuous phase, lb. moles/ cu. ft.
C_p = heat capacity, B.t.u./(lb.)(°F.)
d = impeller diameter, ft.
d_p = particle diameter of dispersed phase, ft.
d_{p50} = particle diameter of dispersed phase at $\phi_D = 0.50$, ft.
D_C = diffusivity of solute in continuous liquid, sq. ft./hr.
D_D = diffusivity of solute in dispersed liquid, sq. ft./hr.
D_D' = effective diffusivity of solute in a liquid drop, sq. ft./hr.
e = 2.7183
$\exp x = e^x$
E_{MC} = fractional Murphree continuous-phase stage efficiency.
E_{MD} = fractional Murphree dispersed-phase stage efficiency.
E_0 = fractional over-all countercurrent cascade stage efficiency.
f = function given by Figure 2, dimensionless.
g_c = conversion factor, (ft./sq. hr.) (lb. mass/lb. force).
h_C = continuous-phase heat-transfer coefficient, B.t.u./(hr.)(sq. ft.) (°F.)
k_C = continuous-phase mass-transfer coefficient, lb. moles/(hr.)(sq. ft.)(lb. moles/cu. ft.); net dimensions ft./hr.
k_H = thermal conductivity, B.t.u. ft./(hr.)(sq. ft.)(°F.)

K = a constant
ln = natural logarithm
n = integer
N = rate of rotation of impeller, 1/hr.
N_P = power number, $Pg_c/\rho N^3 d^5$, dimensionless
N_{Re} = agitator Reynolds number, $d^2 N\rho/\mu$, dimensionless
N_{Sc} = continuous-phase Schmidt number, $\mu_C/\rho_C D_C$, dimensionless
N_{Sh} = continuous-phase Sherwood number, $k_C T/D_C$, dimensionless
N_{toD} = over-all number of dispersed-phase transfer units, dimensionless
N_{We} = Weber number, $d^3 N^2 \rho_C/\sigma$, dimensionless
P = power, ft. lb./hr.
r = radius of a sphere, ft.
s = a constant
t_C = continuous-phase temperature, °F.
t_D = final average dispersed-phase temperature, °F.
$t_D{}^0$ = initial uniform dispersed-phase temperature, °F.
T = diameter of mixing vessel, ft.
T_0 = standard diameter of mixing vessel = 0.5 ft. or equivalent in other units.
v_C = rate of flow of continuous phase, cu. ft./hr.
v_D = rate of flow of dispersed phase, cu. ft./hr.
V = volume of liquid in the mixing vessel, cu. ft.
x = a constant, the mixing index, dimensionless
X = $k_C d_p/2D_D'$, dimensionless
Z = $2k_C\theta/d_p$, dimensionless
θ = average dispersed-phase holding time, hrs.
μ = viscosity of agitated mixture, lbs./ft. hr.
μ_C = viscosity of continuous phase, lbs./ft. hr.
μ_D = viscosity of dispersed phase, lbs./ft. hr.
ν_C = kinematic viscosity of continuous phase = μ_C/ρ_C, sq. ft./hr.
ν_D = kinematic viscosity of dispersed phase = μ_D/ρ_D, sq. ft./hr.
π = 3.1416
ρ = density of agitated mixture, lb./cu. ft.
ρ_C = density of continuous phase, lb./cu. ft.
ρ_D = density of dispersed phase, lb./cu. ft.
$\Delta\rho$ = difference in density of the two liquids, lb./cu. ft.
σ = interfacial tension, lbs. mass/sq. hr. = 28.7 (10^3)(dynes/cm.)
ϕ_C = volume fraction of continuous phase in the mixing vessel
ϕ_D = volume fraction of dispersed phase in the mixing vessel
ψ_n = eigen value

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Flow of Two-phase Carbon Dioxide Through Orifices

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Experimental flow rate data are presented for saturated liquid, saturated vapor, and two-phase liquid-vapor carbon dioxide through a convergent nozzle and a square-edged orifice. The data cover the range from the triple-point pressure to the critical pressure. Charts have been prepared for this complete range at critical flow. Results are also presented for subcritical flow.

The tests at various back pressures indicate that the saturated liquid behaved as a cold liquid without evaporation ahead of the throat.

Saturated vapor became supersaturated in the nozzle, and the vapor behaved as if no condensation occurred.

Equations are presented for the flow rates of saturated vapor, and two-phase mixtures in the critical flow region.

A Mollier (pressure-enthalpy) diagram is used to determine the flow rates of saturated vapor and two-phase mixtures where supersaturation takes place. In these cases, the lines of constant specific volume or density are extrapolated from the superheated region into the normal two-phase region to obtain values corrected for supersaturation.

Critical flow nozzles or orifices are used in many cases to discharge two-phase carbon dioxide from piping systems to the atmosphere. This investigation was undertaken in order to predict the effect of nozzle or orifice design and operating conditions on the flow rate.

Whenever a saturated or boiling liquid flows through a pipeline, the drop in pressure, caused by the flow friction loss, is accompanied by a drop in temperature, and the boiling or evaporation of the liquid to convert a fraction of it to vapor in order to maintain thermodynamic equilibrium. If the liquid is colder than the pipeline, the heat transferred to the liquid also causes the evaporation of part of the liquid. This formation of vapor results in a two-phase mixture of liquid and vapor.

Such a flow of a two-phase mixture of liquid and vapor occurs when liquid carbon dioxide is used for fire extinguishing purposes or for industrial work such as cooling cold chambers or material which is being processed.

HISTORICAL

The flow of saturated liquid and saturated vapor carbon dioxide through nozzles at a back pressure of 142 lb./sq. in. abs. was recently investigated (*10*).

The phenomenon of supersaturation of vapor was disclosed in 1870 by Lord

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